How many acres of potatoes does a society need?

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Abstract. One of the main difficulties in a class on Sources of Energy and Social Policy is the wide variety of units used by different technologists (BTU's, Barrels of oil, Quads, kWh, etc). As every student eats, I think some of this confusion can be resolved by starting and grounding the class with a discussion of food and food production. A general outline for this introduction is provided and two interesting historical cultural examples, Tenichtitlan and the Irish Potato Famine, are provided. Science and Social Policy classes are full of bespoke units and involve many different contexts. Starting the class with a discussion of food energy is a nice way for everyone to start with the same context. In addition, discussion of Food Energy can lead to interesting historical claims.

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1. Introduction

When the United States entered World War One one of the problems they faced was logistics. How much food do you need to ship overseas to Europe to feed a million soldiers? That early work in nutrition led to the 3000 Calorie diet many people remember from secondary Health Education class. A bit about units you might remeber: $1\ Calorie = 1\ kilo - calorie\ (kcal)$, and a dietician might build a $3000 kcal$ diet for a 20 year old basketball player. A *calorie* is the amount of energy it takes to heat a gram of water by a degree Celsius. There are about 4.2 Joules in a single calorie, and a Joule occurs all over introductory physics. If you need to buy a new home furnace, the sales brochure might advertise that it is capable of delivering 100,000 BTU's of heat each hour. What's a BTU? Heat a pound of water by $1°F$. Of course Heat Pumps are far more efficient than simply burning methane or propane, but they consume kilo-watt-hours (kWh) of electricity, not BTU's. What's a kWh? Run a 1000 Watt toaster for an hour and you'll have pulled one kWh off the grid, it will cost you about $0.13 in Minnesota. If you decide to put solar panels in your backyard, they will probably collect about 10% of the 3.5kWh the the sun delivers to each square meter of your lawn (in Minnesota) each day.

As the previous paragraph illustrates, there are a frustratingly large number of different units in an "Energy" class. At Winona State, this 3 credit class fulfulls a "Science and Social Policy" general education requirement and is taken by students from across the university. Lots of college majors don't require a math class beyond algebra or introductory statistics and the population is largely math-averse. You could jokingly say that one of the main things students learn in the class is unit converstion, but it isn't far off. Nearly every field finds energy a useful representation, and every profession has their own set of units and terminology that's most well suited for quick calculation. Would a medical lab scientist talk about the fractional acre-foot of urine needed test kidney function? No, but someone in the central valley of California would certianly care about the acre-feet of water necessary to grow almonds! Does a gas station price their gasoline in dollars per kWh? Given the growing electrification of cars, they might soon.

Everyone eats, maybe not $3000 kcals$ per day, but at least something every day. When I teach our energy class, I spend a few weeks talking about food energy before all other types. While food production is not central to climate change and wars over oil, food is essential in a way that diesel and gasoline are not. Vehicle fuel makes modern life possible, but we could live, unpleasantly, without it. We can't live without fats and protein.

2. Food Energy

To introduce Food Energy, I ask the students to work through a few questions:

2.1. Converting food into body heat

Planning to save money, one college student decides to go to an all-you-can-eat buffet each day at 11am. If he brings homework and stretches the meal out for a few hours he can get all 3000 $kcals$ with only one meal bill. Food is fuel for the human body – could too much fuel make his body feel sick? If his body burned all this food at once, how much warmer would he get? Useful information: the student has a mass of 80kg and is made mostly of water. A Calorie heats 1 kg of water $1°C$.

Here's a possible answer: equate food energy with calorimetric heating and assume human bodies have the same heat capacity as water, about $1\frac{kcal}{kg\cdot°C}$. This allows us to calculate the body's temperature increase.

$$3000kcals = 80kg \cdot 1\frac{kcal}{kg \cdot^{\circ} C} \cdot \Delta T \tag{1}$$

$$\Delta T \approx +37.5°C \tag{2}$$

Students are normally quite surprised at this number. Although wildly unrealistic, $\Delta T \approx +6°C$ is typicaly fatal, there is a related phenomena of diet-induced thermogenesis[1] known informally as "the meat sweats". Some students connect this calculation to feeling quite hungry after a cold swim in the pool (a similar effect). On a larger scale, discussing what's wrong with this estimate is useful. The main storage mechanism for storing food energy is fat tissue, which the calculation completely ignores. Infants are generally born with little fat, and an infant sleeping through the night often coincides with the baby growing enough fat tissue to store sufficient kcals to make it though a night without waking up ravenously hungry. A related follow-up is that if a person is stranded in the wilderness, they should immediately start walking downstream (ie, towards civilization) as they likely won't be able to harvest an amount of kcals equivalent to what they already have stored on their hips and abdomen.[2] The contrast of bear hibernation [3] and songbirds constatly eating through the winter are related connections to investigate.

2.2. Biophysical Power

A more realistic question to follow up with relates to the average *power* given off by a person over a day. Again, assuming $3000kcal$ is burned over $24hours$, with useful information: $1kcal \approx 4200J$ and $1J/s = 1W$.

$$\frac{3000kcal}{24hours} \cdot \frac{4200J}{1kcal} \cdot \frac{1hour}{3600sec} \approx 145W \tag{3}$$

Most students still remember $75Watt$ lightbulbs, but given the spread of LED lighting, "A person's body heat is two 75W light bulbs" will probably only make sense for a few more years. Desert or cold-weather camping, alone versus with friends, and survival swimming are also examples for students to make sense of this answer. If you can take advantage of other people's waste body heat, you'll sleep more pleasantly and survive longer in cold water.

Another application to discuss is that of “brown fat,” a sort of biological space heater that humans and other mammals develop in response to cold weather. This tissue’s mitochondria can burn lipids and carbohydrates in a useless proton pumping scheme, which produces metabolic heat [4]. Most common in rodents and infants, this mechanism can be stimulated by extended exposure to cold temperatures. The idea of a biological space heater that takes a month to turn on and a month to turn off matches the lived experience of college students in Minnesota, who wear down jackets in $4°C$ weather in November, and beachwear in the same $4°C$ weather in March. Additionally, transplants to northern climates often take a few years to “get used to” the colder weather up north. It seams just as easy to say that transplants’ bodies take a few years to develop the brown fat cells which allow them to be comfortable in cold weather.

One other distinction to emphasize is the difference between power and energy. A graph of a human body’s “kcal content” over the course of a day can be a useful illustration. When sedentary, this graph probably has the slope of $-150W \approx -125\frac{kcals}{hour}$. If the $3000kcal$ meal at the buffet takes an hour, this period corresponds to an energy-time slope of $+3000\frac{kcal}{hour} \approx +3500W$.

In medicine, these slopes are effectively equivalent to “Metabolic Equivalent of Task” (METS), a common measure in cardiology and exercise physiology. METS is power normalized by mass, $1METS = 1\frac{kcal}{kg \cdot hour}$, and METS levels are available for many different physcial activities. [5]

2.3. Burning off food energy

Imagine that after eating a 600 $kcal$ bacon-maple long-john (donut), you decide to go for a hike to “work off” the Calories. Winona State is in a river valley bounded by $200m$ tall bluffs. How high up the bluff would you have to hike to burn off the donut? Useful information: human muscle is about 1/3 efficient, and on Earth’s surface, gravitational energy has a slope of about 10 $\frac{Joules}{kg \cdot m}$.

One way to approach this problem is by using Energy Bar Charts [6] to illustrate how the energy held in food changes form as it is used. An approximation for this question is shown in figure 1. In this story, the “system” is taken to be the earth, food, and hiker. The hiker’s body is assumed to be 1/3 efficient, which means one of the food energy blocks of energy is transformed into gravitational energy (elevation) at the end of the hike. The other 2 blocks of energy are transformed into heat and leave the hiker’s body, most likely by mechanisms of respiration and sweat evaporation. The purpose of a bar chart like this is to provide a pictoral and mathematical representation of the energy conservation equation given in 4.

$$\begin{aligned} \frac{1}{3} \cdot 600kcal \cdot \frac{4200J}{1kcal} &= 80kg \cdot 10\frac{Joules}{kg \cdot m} \cdot height && (4) \\ height &\approx 1000m && (5) \end{aligned}$$

This estimate is again surprising to students. Five trips up the bluff to burn off \$2 of

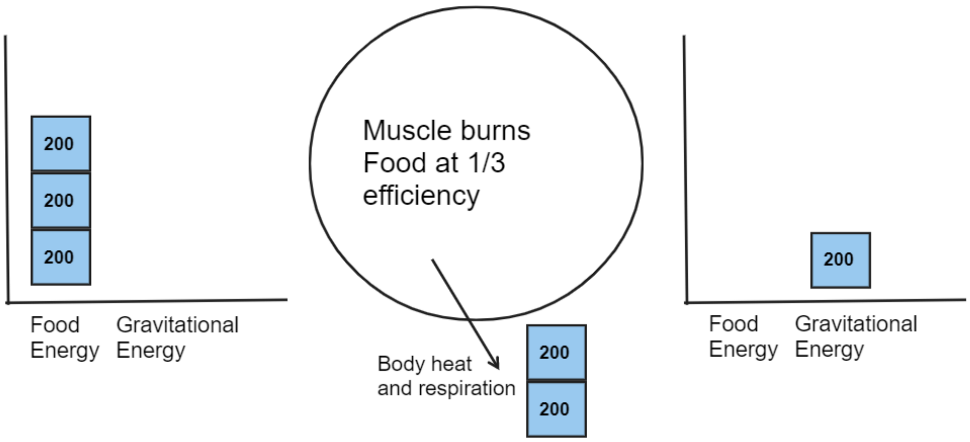


Figure 1. An Energy Bar Chart to illustrate the 1/3 efficient student hiking up a bluff to burn off the morning's donut. The initial state (left) is the hiker at the bottom of the hill, with donut in stomach. The final state (right) is the hiker at the top of the bluff with 2/3 of the energy removed to the atmosphere by sweat and exhalation of warm air. 1/3 of the donut's energy is stored in elevation. The system for this diagram includes the earth, the hiker, and the donut. The system does not include the atmosphere around the hiker.

saturated fat, sugar, and flour! A nice followup calculation is to imagine a car that can burn a $100kcal$ piece of toast in the engine: from rest, what speed will the toast propel it to? If (again) the engine converts 1/3 of the energy into motion (kinetic energy), a 1300kg Honda Civic will reach a speed of about $13\frac{m}{s} \approx 33mph$!

The point of these energy calculations is not to give students an eating disorder. Rather, the numbers show food's amazing power. A single slice of toast will bring a car up to the residential speed limit! A day's food, $3000kcal$, will power you up an $8000m$ mountain peak! The body-work food allows us to do is astonishing, and increases in food production have made modern comforts, unimaginable 150 years ago, possible to the point of being taken for granted.

2.4. Where does food energy come from?

One feature of the aught's "homesteading" culture [8] is the idea that a person should probably be able to move to the country, eat a lot of peaches, and grow all their own food. Learning that farming labor is *skilled* labor can be a brutal and disheartening realization. Eating $3000kcals$ each day means planting, weeding, harvesting, and storing more than a million kcals each year [7]. Where will those Calories come from? Is your backyard enough to homestead in the suburbs [9]?

At some point bewteen 1920 and 1950, US chemical manufacturers realized that in the post-war period, they could repurpose processes developed for manufacturing munitions and chemical warfare agents, to produce chemicals that would kill insects and increase the nitrogen levels in the soil. As figures 2 and 3 show, the epoch of "Better Living Through Chemistry" produced a dramatic increase in per-acre yields across all comodity food crops, particularly corn and potatoes.

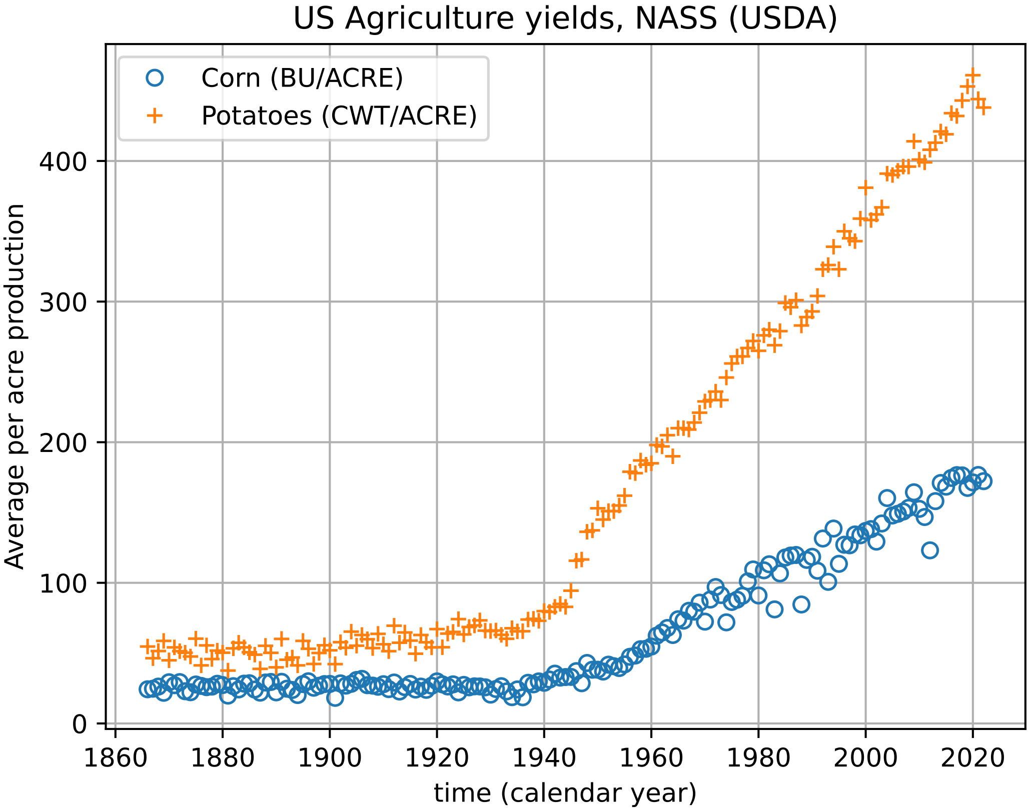


Figure 2. USDA per acre Corn and Potato production figures, plotted over time. Data is given in harvet units, $56lbs$ bushels per acre for field corn and hundred-weight (CWT) for potatoes. By mass, corn is about 4.5 times more calorie dense than potato which results in a nearly equal $kcal/acre$ values for both crops in figure 3. Details on the data source and conversions are given in Appendix A.

However, if you're discussing backyard Calorie production it isn't reasonable to use modern yield estimates for planning. "Roundup Ready" Corn, Soybean, and Sugar Beet seeds are not available to the public, nobody wants to put on a respirator to apply Atrazine ten feet from the back door, and the edge effects from deer and insects are much smaller on a 600 acre field than they are in an community garden allotment. As mentioned in the introduction, in 1917 the USDA published a pamphlet [10] giving detailed Calorie estimates a farmer might expect from a given acre of a crop. A table from this pamphlet is shown in Figure 4. The pamphlet data came from pre-war, pre-chemical agriculture, and the yields cited were produced with horses, manure, lime, and

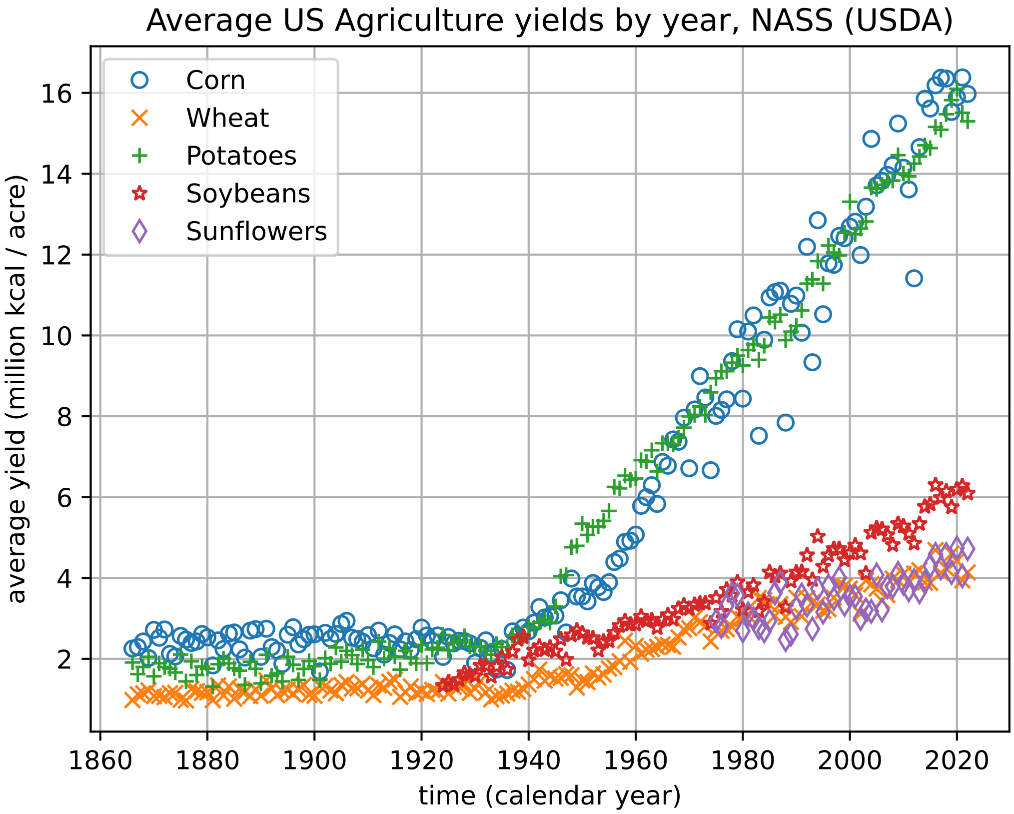


Figure 3. USDA per acre crop production figures, plotted over time. Production data is scaled by estimated dietary kcal content to show that, over all crops, there has been a dramatic increase in kcal production since about 1940. Details of the data source and conversions are given in Appendix A. The idea for this plot came from an online blog, [11]. It would be interesting to know if there are patterns of scaling among vegetable families (grains, legumes, tubers, etc) in the same way that there are family classifications for the minimal energy required for transport [12].

large families full of children. If you want to be self sufficient, these yield numbers are probably a good upper bound on what's realistically possible by a dedicated luddite.

So, another question using this data. If you want to feed your family of four people potatoes, how much land will you need to cultivate? Here's an estimate: a family of 4 requires 3000kcal/person each day. If we over-estimate and produce food for the entire year, the family will need about 4.4 million kcals.

$$4 \; people \cdot \frac{3000kcal}{person \cdot day} \cdot \frac{365 \; days}{year} \approx 4.4Mkcal \tag{6}$$

A brief aside for those bored by the simplistic unit conversion: when I ask students to solve problems like these, one undercurrent of conversation is “Should I divide by 365 or multiply?” Particularly with online homework systems, checking your answer for reasonability isn't typically graded. Asking the students to reason proportionaly with units is a skill that can give meaning to numbers.

From figure 4 we can estimate 1.9 million kcals per acre of potato production. Again the students might ask, should I multiple 4.4 and 1.9 or should I divide them. It can be useful in a class discussion to have the students discuss and vote which of the following two forms will give the meaningful answer.

$$\frac{4.4Mkcal}{family} \cdot \frac{1acre}{1.9Mkcal} \quad \text{or} \quad \frac{4.4Mkcal}{family} \cdot \frac{1.9Mkcal}{1acre} \tag{7}$$

The choice of operation is difficult to make without seeing the units present, which is again a learning opportunity for the students.

What does the answer of 2.3 acres mean? The university's $91m \times 49m$ football field has an area of about 1.1 acres, so you could say that a football field planted in potatoes will probably feed a family through the winter. Can a person enjoy the benefits of urban living and grow all their own food? The population density of New Jersey is $1,263\ people/mile^2 \approx 1.97\ people/acre$ and our 4 person family needs 2.3 acres for their potatoes. Unless the social model is one of a country Dacha or an endless suburb with no duplexes or apartment buildings, urban living and food self-sufficiency seem mutually exclusive.

More emotionally charged conversations can be had about converting the United States to all organic agriculture, which typicaly has a yield penalty of about $20-40bu/acre$ when compared to conventional production. The 1917 data isn't directly applicable, but it relates. At $180bu/acre$ conventional corn requires 22 million acres (half of Wisconsin, or all of Indiana) to feed the US population (350 million people) corn for a year. The remainder of the corn belt can be devoted to animal feed, ethanol, and export. If the corn belt was devoted to producing organic corn at lower yield [13], we probably wouldn't starve, but cheap meat and ethanol vehicle fuel would likely disappear.

3. Example: How big could Tenochtitlan have been?

The questions described thus far have largely been centered within a physics context. The paper closes with two more examples that leverage this food energy picture to make historical claims. The first example relates to the pre-columbian capital of the Aztec Empire, Tenochtitlan, now known as Mexico City. Tenochtitlan was build on and around a endorheic lake, Texcoco. Crops were grown in shallow parts of the lake via chinampas, floating patches of decaying vegetation and soil. Given the proximity to water and decaying vegetation, these fields were very fertile and productive.

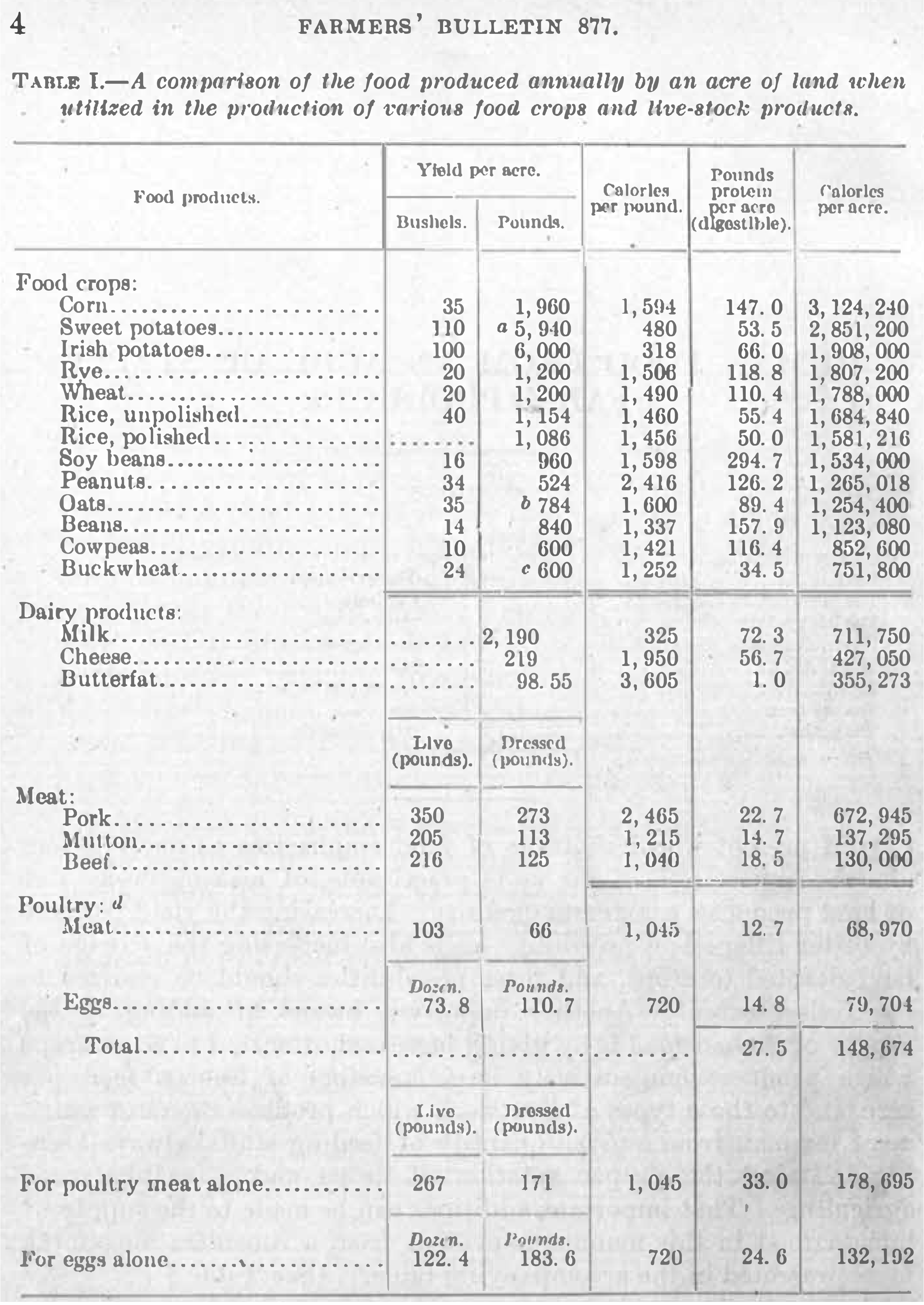
Estimates of Tenochtitlan's population in 1500CE vary widely, from 40,000 [14] to more than 400,000 [15] inhabitants, comparable in size to Paris at that time. These estimates come from oral and written records and estimates of archeological building density and land area.

4. Example: Was the Irish Potato Famine a Natural Disaster?

5. Conclusion

Acknowledgments

The work was influenced by discussions with John Deming, Carl Ferkinhoff, and Sarah Taber.



4 FARMERS' BULLETIN 877.

TABLE I.—*A comparison of the food produced annually by an acre of land when utilized in the production of various food crops and live-stock products.*

Food products.	Yield per acre.		Calories per pound.	Pounds protein per acre (digestible).	Calories per acre.
	Bushels.	Pounds.			
Food crops:					
Corn	35	1,960	1,594	147.0	3,124,240
Sweet potatoes	110	[a] 5,940	480	53.5	2,851,200
Irish potatoes	100	6,000	318	66.0	1,908,000
Rye	20	1,200	1,506	118.8	1,807,200
Wheat	20	1,200	1,490	110.4	1,788,000
Rice, unpolished	40	1,154	1,460	55.4	1,684,840
Rice, polished		1,086	1,456	50.0	1,581,216
Soy beans	16	960	1,598	294.7	1,534,000
Peanuts	34	524	2,416	126.2	1,265,018
Oats	35	[b] 784	1,600	89.4	1,254,400
Beans	14	840	1,337	157.9	1,123,080
Cowpeas	10	600	1,421	116.4	852,600
Buckwheat	24	[c] 600	1,252	34.5	751,800
Dairy products:					
Milk		2,190	325	72.3	711,750
Cheese		219	1,950	56.7	427,050
Butterfat		98.55	3,605	1.0	355,273
	Live (pounds).	Dressed (pounds).			
Meat:					
Pork	350	273	2,465	22.7	672,945
Mutton	205	113	1,215	14.7	137,295
Beef	216	125	1,040	18.5	130,000
Poultry: [d]					
Meat	103	66	1,045	12.7	68,970
	Dozen.	*Pounds.*			
Eggs	73.8	110.7	720	14.8	79,704
Total				27.5	148,674
	Live (pounds).	Dressed (pounds).			
For poultry meat alone	267	171	1,045	33.0	178,695
	Dozen.	*Pounds.*			
For eggs alone	122.4	183.6	720	24.6	132,192

Figure 4. A table from a USDA booklet giving 1917 yields for various farm products.

Table A1. A summary of units and conversions used to create figure 3 from USDA NASS data. $1cwt$ is a hundred pounds of potatoes. A bushel, $1bu$, is a volume unit of about 35liters and corresponds to about 60lbs of grain. Calorie content per 100 gram mass of food is taken from the USDA's "Food Data Central" database. It isn't clear from any of these resources if lb is pound-force (lbf) or pound-mass (lbm) and so I am ignorantly treating them as "grocery store units" where $1lbs = 453.592grams$.

Crop	per acre unit	production unit	kcals per 100gram	FDC ID
Corn	bu/acre	$1bu = 56lbs$	365	170288
Potatoes	cwt/acre	$1CWT = 100lbs$	77	170026
Soybeans	bu/acre	$1bu = 60lbs$	446	174270
Sunflowers	lbs/acre		584	170562
Wheat	bu/acre	$1bu = 60lbs$	327	168890

Appendix A. Creating the historical kcal/acre figure from USDA data

The United States Department of Agriculture (USDA) provides historical crop information via the National Agricultureal Statistics Service, `https://www.nass.usda.gov/Statistics_by_Subject/index.php?sector=CROPS`. Data was downloaded in spreadsheet csv format and then combined and plotted via a Python Jupyter notebook.

Each crop has its own bespoke units, for example potatoes are sold by hundredweight (CWT) but sugar beets are measured by the ton. Every imaginable agricultural product seems to be tracked in the NASS site, for example Maple Syrup production is tracked and given in gallons of syrup per (tree) tap! Conversion factors used are summarized in Table A1. Calorie (kcal) density for each crop was taken from `https://fdc.nal.usda.gov/fdc-app.html`. Within this database, foods are identified by an FDC ID.

An example calculation (implemented in the Jupyter notebook) follows for Corn. In 2022 the USDA reported an average production of 172.3 bushels of corn per acre of farmland.

$$172.3\frac{bu}{acre}\cdot\frac{56lbs\ corn}{bu}\cdot\frac{453.592\ grams}{lbs}\cdot\frac{365\ kcal}{100\ grams} = 15,974,657\frac{kcal}{acre} \tag{A.1}$$

Obviously the result is only reasonable to two signifigant figures!

Raw data from the USDA NASS is plotted in figure A1. The scaling described in equation A.1 produces figure 3 earlier in the paper.

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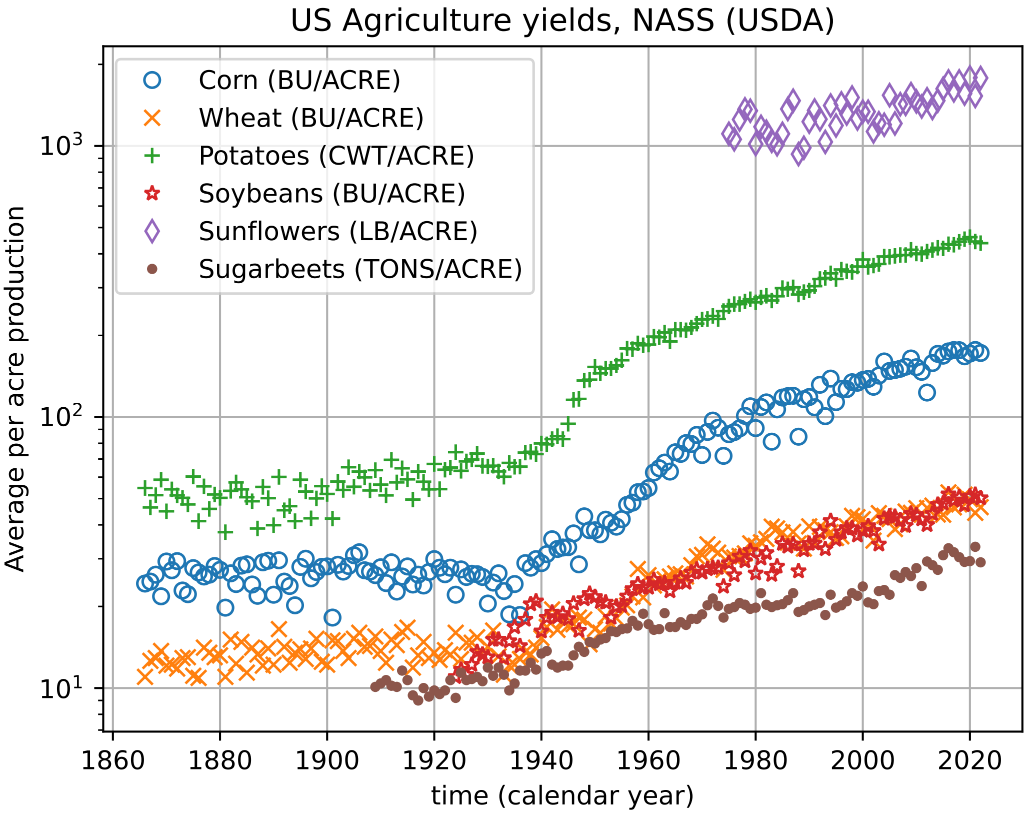

Figure A1. USDA yields from pre-chemical US ag

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